CBCS SCHEME

USN

15MAT41

Fourth Semester B.E. Degree Examination, Dec.2023/Jan.2024
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 80

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. Use of statistical table is permitted.

Module-1

1 a. Using the Taylor's series method, solve $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$ at the point $x = 0.1$ (upto 3rd degree term). **(05 Marks)**

b. Use Runge-Kutta method to find y at x = 1.1 given that, $\frac{dy}{dx} = x^2 + y^2$, $y(1) = 1.5$. **(05 Marks)**

c. Apply Milne's predictor-corrector method to find y(0.8) given that, $\frac{dy}{dx} = x - y^2$, $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$. Carry out two corrections for the solution. **(06 Marks)**

OR

2 a. Using the modified Euler's method, solve $\frac{dy}{dx} = x + y$, $y(0) = 1$ at $x = 0.2$. Take $h = 0.2$ and carry out two modifications. **(05 Marks)**

b. Using the 4th order Runge-Kutta method, find y(0.2) given that $\frac{dy}{dx} = \frac{y - x}{y + x}$, $y(0) = 1$ **(05 Marks)**

c. Find y(0.8) given that $\frac{dy}{dx} = x + y$, $y(0) = 1$, $y(0.2) = 1.2$, $y(0.4) = 1.48$, $y(0.6) = 1.856$ by using Adams-Bash forth method. **(06 Marks)**

Module-2

3 a. Using Runge-Kutta method find y(0.2) given that $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$, $y(0) = 1$ and $y'(0) = 0$ taking $h = 0.2$. **(05 Marks)**

b. Express $f(x) = x^3 + 2x^2 - x - 3$ in terms of Legendre polynomials. **(05 Marks)**

c. If α and β are distinct roots of the equation $J_n(x) = 0$ then prove that, $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0$. **(06 Marks)**

OR

4 a. Using Milne's method find y(0.8) given that $\frac{d^2y}{dx^2} = 2y\frac{dy}{dx}$, y(0) = 0, y(0.2) = 0.203, y(0.4) = 0.423, y(0.6) = 0.684, $y'(0) = 1$, $y'(0.2) = 1.041$, $y'(0.4) = 1.179$, $y'(0.6) = 1.468$ **(05 Marks)**

b. Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$. **(05 Marks)**

c. Prove the Rodrigues formula, $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}\left[(x^2-1)^n\right]$. **(06 Marks)**

Module-3

5 a. State and prove Cauchy-Riemanna equations in Cartesian form. **(05 Marks)**

b. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}dz$, where C is $|z| = 3$. **(05 Marks)**

c. Find the bilinear transformation that transforms the points $z_1 = 0$, $z_2 = -i$, $z_3 = -1$ on to the points $w_1 = i$, $w_2 = 1$, $w_3 = 0$. **(06 Marks)**

OR

6 a. State and prove Cauchy's theorem. **(05 Marks)**

b. Discuss the transformation $w = e^z$. **(05 Marks)**

c. Find the analytic function $f(z) = u + iv$ given that $u = e^{2x}(x\cos 2y - y\sin 2y)$. **(06 Marks)**

Module-4

7 a. The probability density function of a random variable X is given by,

x :	0	1	2	3	4	5	6
P(x) :	K	3K	5K	7K	9K	11K	13K

Find : (i) K (ii) $P(x < 4)$ (iii) $P(x \geq 5)$ **(05 Marks)**

b. Derive mean and variance of Poisson distribution. **(05 Marks)**

c. A joint probability distribution is given by,

x \ y	-3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

Find E(x), E(y), E(xy). **(06 Marks)**

OR

8 a. The probability that a pen manufactured by a company will be defective is 0.1. If 12 such pens are selected find the probability that,
(i) Exactly 2 will be defective.
(ii) Atleast 2 will be defective
(iii) None will be defective. **(05 Marks)**

b. The mean weight of 500 students at a certain school is 50 kgs and standard deviation is 6 kgs. Assuming that the weights are normally distributed, find the probability that the students weighing (i) between 40 kgs and 50 kgs
(ii) More than 60 kgs [A(1.667) = 0.4525]. **(05 Marks)**

c. The marginal distribution of two independent random variables x and y are given by,

x	0	1
f(x)	0.2	0.8

y	1	2	3
g(y)	0.1	0.4	0.5

Find joint probability distribution of x and y and also cov(x, y). **(06 Marks)**

Module-5

9 a. The mean and standard deviation of marks scored by a sample of 100 students are 67.45 and 2.92. Find 95% confidence interval. **(05 Marks)**

b. Consider the sample consisting of nine numbers 45, 47, 50, 52, 48, 47, 49, 53 and 51. The sample is drawn from the population whose mean is 47.5. Find whether the sample mean differs significantly from the population mean at 5% level of significance $[t_{0.05}(8) = 2.31]$. **(05 Marks)**

c. A student's study habits are as follows : If he studies one night, he is 60% sure not to study the next night, on the other hand if he does not study one night, he 80% sure to study the next night. In the long run how often does he study? **(06 Marks)**

OR

10 a. The mean life-time of a sample of 100 fluorescent tube lights manufactured by a company is found to be 1570 hours with a standard deviation of 120 hours. Test the hypothesis that the mean life-time of the lights produced by the company is 1600 hours. **(05 Marks)**

b. Define the following :

(i) Null hypothesis.
(ii) Type-I and Type-II errors.
(iii) Regular stochastic matrix.
(iv) Fixed probability vector. **(05 Marks)**

c. Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C. But C is just as likely to throw the ball to B as to A. If C was the first Person to throw the ball, find the probabilities that for the fourth throw :

(i) A has the ball
(ii) B has the ball
(iii) C has the ball. **(06 Marks)**

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